Written Exam at the Department of Economics summer 2020-R

Industrial Organization

Final Exam

14 August 2020

(5-hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 3 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Collusion between Stackelberg competitors

Consider a market in which two firms (indexed by $i = 1, 2$) are competing in quantities. The market price is determined by the inverse demand function $p = 2 - (q_1 + q_2)$, where q_i is firm i's output. Each firm i has a constant per-unit cost, denoted by $c_i \in [0, 2)$. Firm's profit can thus be written as

$$\pi_i = (2 - c_i - q_1 - q_2)q_i. \tag{1}$$

To start with, assume that the two firms interact in a one-shot game. Moreover, the firms move sequentially, à la Stackelberg: first, firm 1 chooses its output $q_1 \geq 0$; thereafter, firm 2 observes q_1 and then chooses its own output $q_2 \geq 0$. Each firm's objective is to maximize the own profit, as given by (1).

(a) Solve for the subgame perfect Nash equilibrium of the Stackelberg game described above.

A pair of output levels is said to be *Pareto efficient* if there exists no other pair of output levels that makes at least one firm strictly better off without making the other firm worse off.

Formally, (q_1, q_2) is Pareto efficient if there exists no (q_1', q_2') such that $\pi_i(q_1', q_2') \geq \pi_i(q_1, q_2)$ for both $i = 1$ and $i = 2$, with at least one of the inequalities being strict.

(b) For what values of c_1 and c_2 is the outcome of the equilibrium that you found in part (a) Pareto efficient? Prove your answer formally.

Now assume that there are infinitely many and discrete time periods t (so $t = 1, 2, 3, \ldots$) and that, at each t, the firms play the Stackelberg game described above, choosing their respective output levels $q_{it} \geq 0$. The firms' common discount factor is denoted by $\delta \in [0, 1)$. At the end of each time period, firm 1 can observe firm 2's choice of q_{2t} (in addition, the above description of the Stackelberg game told us that firm 2 observes q_{1t} prior to choosing q_{2t}). To simplify the model, assume that the cost parameters are given by

$$(c_1, c_2) = (0, 0).$$

Let a pair of collusive output levels be given by $(q_1^c, q_2^c) = (1 - \lambda, \lambda)$, where $\lambda \in [0, \frac{1}{2}]$ is a constant. Consider the following grim trigger strategy: In period $t = 1$, firm 1 chooses $q_{11} = q_1^c$. After that, whenever it is firm i's turn to make a choice:

- firm i chooses $q_{it} = q_i^c$ if each firm i chose q_i^c on all previous occasions;
- otherwise (so if at least one of the two firms chose some $q_{it} \neq q_i^c$ on any previous occasion), $q_{it} = q_i^S$.

Here, q_i^S is firm i's equilibrium output level in the one-shot version of the game (so the output level you were asked to identify in part (a)).

(c) Investigate under what conditions the two firms' following the above trigger strategy constitutes a subgame perfect Nash equilibrium of the infinitely repeated game. In particular, derive a (necessary and sufficient) condition for firm i (for $i = 1, 2$) not to have an incentive to deviate from the strategy (given that the other firm follows it). Each condition should be stated as $\delta \geq K_i$, where K_i is function only of λ.

Question 2: Price discrimination and Cournot competition in a vertically related market

In the country of Rainlandia, raincoats are produced by a monopoly firm called U. All raincoats that are produced by U are sold to the final consumers by two retailers, called D_1 and D_2.

We model the interaction between the retailers as a Cournot game. In particular, each retailer i (for $i = 1$ and $i = 2$) chooses its output $q_i \geq 0$, and then the market price is determined by the inverse demand function $p = 1 - Q$, where $Q \stackrel{\text{def}}{=} q_1 + q_2$. If a retailer chooses the output q_i, then it must pay the amount $w_i q_i$ to U, where w_i is the per-unit wholesale price. In addition, each retailer must incur a constant cost c_i for each unit that it sells. We assume that $0 \leq c_1 \leq c_2 < 1$; moreover, the difference between c_1 and c_2 is small enough to ensure that, at the equilibrium of the model, the constraints $q_i \geq 0$ do not bind. The profit of retailer D_i can be written as $\pi_i = (1 - w_i - c_i - Q)\, q_i$. The upstream firm U is assumed not to have any production costs and its profit can therefore be written as $\pi_U = w_1 q_1 + w_2 q_2$.

The timing of events is as follows.

(i) The upstream firm U chooses w_1 and w_2.

(ii) The retailers D_1 and D_2 observe both w_1 and w_2, and then they simultaneously and independently choose their own output q_i.

Each firm's objective is to maximize the own profit.

(a) Solve for the subgame perfect Nash equilibrium values of q_1 and q_2.

In the model above, the downstream firms could be charged separate wholesale prices, w_1 and w_2; that is, the upstream firm was able to practice price discrimination. Suppose now instead that there is a ban on price discrimination, meaning that $w_1 = w_2 = w$. All other parts of the model are the same as before.

(b) In this new game with a ban on price discrimination, solve for the subgame perfect Nash equilibrium values of q_1 and q_2.

(c) Do/answer the following:

(i) Compare the consumer surplus ($CS = [(q_1 + q_2)^2]/2$) with and without price discrimination. Are the consumers (according to this measure) better or worse off from a ban on price discrimination?

(ii) Compare the industry profits ($\Pi = \pi_U + \pi_1 + \pi_2$) with and without price discrimination. Are the firms jointly better or worse off from a ban on price discrimination?

You are encouraged to answer part (iii) also if you have failed to solve part (ii).

(iii) What is the logic behind your result under (ii)? Discuss!

End of Exam